Mat 2377: Quiz #1

May 11, 2016

Answer all the questions first in one 45 minute sitting. Then check your answers against the colutions given below.

1. If $P(A) = 0.4, P(B) = 0.5$ and $P(A \cap B) = 0.3$, find $P(A \cap B')$

2. What is the probability of getting no clubs in a hand of 5 cards in the game of poker?

3. A coffee shop lets you order a snadwich with your choices of bread, meat, cheese and condiments. There are 6 choices for bread, 4 choices for meat, 4 choices for chees and 12 choices for condiments. How many possible sandwiches are there if you choose one bread, one meat, one cheese and one condiment?

4. At a fair, a vendor has 25 helium balloons on strings: 10 are yellow, 8 are red and 7 are green. A balloon is selected at random and sold. Given that the balloon sold is yellow, what is the probability that both selected balloons are yellow?

5. Four cards are to be dealt successively at random and without replacement from a deck of 52 cards. What is the probability of receiving in order a spade, a heart, a diamond and a club?

6. A grocery store had 10 cartons of milk, two of which were sour. If you are going to buy the sixth carton of milk that day chosen at random, what is the probability of selecting a carton of sour milk?

7. Let A, B be two independent events with $P(A) = 0.7, P(B) = 0.2$. Compute $P(A' \cup B')$.

8. Each bag in a large box contains 25 tulip bulbs. Three-fourths of the bags contain bulbs for 5 red and 20 yellow tulips; one fourth of the bags contain bulbs for 15 red and 10 yellow tulips. A bag is selected at random and one bulb is planted. a) What is the probability that it will produce a red tulip? b) If the tulip is red, what is the probability that a bag containing 5 red and 20 yellow tulips was selected?

Solutions

1. $P(A \cap B')$=$P(A) - P(A \cap B) = 0.4 - 0.3 = 0.1$

2. We conside rthe complementary event, that of all cards being clubs.

$$P(noclubs) = \frac{\binom{39}{5}}{\binom{52}{5}} = \frac{575757}{2598960} = 0.22153$$

3. $6 \times 4 \times 4 \times 12 = 1152$

4. Let A_1 be the event that the first selected balloon is yellow and let A_2 be the event that the second selected balloon is yellow. We have that

$$P(A_1 \cap A_2) = P(A_2|A_1) P(A_1) = \left(\frac{9}{24}\right)\left(\frac{10}{25}\right) = \frac{3}{20}$$

5. $\left(\frac{13}{52}\right)\left(\frac{13}{51}\right)\left(\frac{13}{50}\right)\left(\frac{13}{49}\right) = \frac{2197}{499800} = 4.3958 \times 10^{-3}$

6. $\frac{2}{10}$

7. $P(A' \cup B').= 1 - P(A \cap B) = 1 - P(A) P(B) = 1 - 0.7(0.2) = 0.86$

8. We use Bayes theorem. Let B be the event that the bag chosen contains 5 red and 20 yellow tulips and let A be the event of having a red tulip. Then, a)

$$\begin{aligned} P(A) &= P(A|B) P(B) + P(A|B') P(B') \\ &= \frac{5}{25}\frac{3}{4} + \frac{15}{25}\frac{1}{4} = \frac{3}{10} \end{aligned}$$

b) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{5}{25}\frac{3}{4}}{\frac{3}{10}} = \frac{1}{2}$